The topic of our third meeting this semester is **the four colour theorem** - sometimes called **the four colour map theorem**. It states that to colour a map such that no two adjacent regions have the same colour, four colours are sufficient.

Let us make this a bit more precise:

- A map is a separation of a plane into contiguous rigions.
- A region is a connected open subset of the plane whose boundar consists of finitely many straight lines.
- Regions are adjacent, if they share a boundary segment.
- Regions that share only isolated boundar points are not adjacent.

The four colour problem was first proposed in the middle of the 19th century by the brothers Francis and Frederick Guthfrie, students of Augustus De Morgan (a famous British logician, known for 'De Morgan's law'). At the time, one of the brothers was colouring a map of England and notivced that apparently only four colours were needed.

This seems a simple enough fact. And proofs for it soon emerged - which were all exposed as false. Until Kenneth Appel and Wolfgang Haken at the University of Illinois announced a proof in 1979. The particular about this proof was that it was the first proof of a major theorem achieved with extensive computer assistance.

As always, I encourage you to take up your own research beforehand, so that we can have a lively conversation. Any contribution is welcome, be it a specific question, a worked out example, a technical detail that you find interesting, a mathematical fact that is surprising, considerations about society and security,...
It doesn't have to be perfect, simply let your curiosity and creativity lead you.

Here are some suggestions for you to start:

(a) The four colour theorem can be reformulated in terms of graphs. Can you explain how this is done? Maybe give some examples of maps and the associated graphs.
`http://web.math.ucsb.edu/~padraic/ucsb_2014_15/ccs_discrete_w2015/ccs_discrete_w2015_lecture4.pdf`

(b) The unusual nature of the proof was a source for criticism. What are some of the reasons?
`https://math.stackexchange.com/questions/23409/did-the-appel-haken-graph-colouring-four-colour-`
`https://sites.math.rutgers.edu/~zeilberg/Opinion51.html`
`http://www.justinmullins.com/four_colour_theorem.htm`

(c) The proof often comes up when the following question is discussed: What is good mathematics? Indeed, what **is** good mathematics?
`https://arxiv.org/abs/math/0702396`
`https://arxiv.org/abs/math/9404236`

(d) Here is a nice overview of ideas that occurred during attempted proofs.
`https://nrich.maths.org/6291`

- Can you explain the five neighbour conjecture?
- What are minimal criminals?
- What is the six colour theorem?
- What are snarks?
- What does the four colour problem has to do with Euler's formula?

(e) There are some ideas out there for more elegant proofs of the four colour problem.
`https://researchoutreach.org/articles/an-elegant-proof-of-4-colour-theorem/`
`https://rjlipton.wordpress.com/2009/04/24/the-four-color-theorem/`

(f) Here is a nice exhibition of another computer based proof of the four colour problem, that simplifies the proof by Appel and Haken.
`http://people.math.gatech.edu/~thomas/FC/fourcolor.html`

(g) There are many related problems out there. One is the problem about the chromatic number of the plane.
https://www.quantamagazine.org/decades-old-graph-problem-yields-to-amateur-mathematician-201804
Can you describe the problem? What is the chromatic number of a graph?
https://www.quantamagazine.org/the-numbers-and-geometry-behind-a-math-coloring-puzzle-20180618/
(You could use any of the two reference as a basis short presentation.)

(h) What are applications of the four colour theorem?
https://cs.stackexchange.com/questions/22892/application-of-the-four-color-theorem

(i) There are some mathematicians who think that computers will help us proof theorems in the future, some say this will be unavoidable. https://www.imperial.ac.uk/people/k.buzzard
One such project to make this happen is LEAN.
https://leanprover.github.io/
The community edition is a good way to get started: https://leanprover-community.github.io/
Maybe this would be a good future topic for us...