

The topic of our fourth meeting is knot theory – the study of closed curves in three dimensions, and their possible deformations without one part cutting through another. While inspired by knots which appear in daily life, such as those in shoelaces and ropes, a mathematical knot differs in that the ends are joined together so that it cannot be undone. Knot theory, in essence, is the study of the geometrical aspects of the shapes of knots. Not only has knot theory developed and grown over the years in its own right, but also it has been shown to have applications in various branches of the sciences, for example, physics, molecular biology, chemistry, et cetera.

There are two important questions in knot theory:

- Is a closed curve truly knotted or can it simply be untangled? That is, can it be deformed in space into a standard unknotted curve like a circle?
- More generally, represent any two given curves different knots or are they really the same knot in the sense that one can be continuously deformed into the other?

We are very happy to welcome Lukas Lewark as our guest speaker who is an expert on this topic.

Here are some suggestions, remarks and resources:

- (a) A very nice short introductory video:
<https://www.youtube.com/watch?v=WXVItJnZiWc>
<https://www.youtube.com/watch?v=M-i9v9VfCrs>
- (b) This page gives a nice introduction into the fundamentals of knot theory:
http://pi.math.cornell.edu/~mec/2008-2009/HoHonLeung/intro_knots.htm
- (c) Here is a short introduction to knot theory by Giovanni De Santi: <https://graphics.stanford.edu/courses/cs468-02-fall/projects/desanti.pdf>
- (d) There are many books that discuss and introduce knot theory. If you are interested take a look at Murasugi's very nice book:
<https://www.maths.ed.ac.uk/~v1ranick/papers/murasug3.pdf>
 By the way, the above pdf is posted on Andrew Ranicki's homepage who was the son of Marcel and Teofilia Reich-Ranicki.
- (e) He collected some useful links and resources on knot theory here:
<https://www.maths.ed.ac.uk/~v1ranick/knots/>
- (f) Here is a nice survey of knot theory and its applications by Akveld–Neumaier:
<https://people.math.ethz.ch/~akveld/ArtikelAkveldNeumaier.pdf>
- (g) What are applications in knot theory?
 On the following mathoverflow-thread you can find some interesting answers about actual and hypothesised applications of knot theory:
<https://mathoverflow.net/questions/48222/applications-of-knot-theory>
 For example: how do knots appear in quantum theory?
<https://www.ias.edu/ideas/2011/witten-knots-quantum-theory>
- (h) The knot atlas: How can knots be represented and classified?
http://katlas.math.toronto.edu/wiki/Main_Page
- (i) What are some knot invariants?
 Check out this table by Charles Livingston: <https://knotinfo.math.indiana.edu/>
- (j) Knotplot is an elaborate program to visualize and manipulate knots: <https://knotplot.com/>
<https://knotplot.com/knot-theory/>
 For example how to untie the unknot: <https://www.youtube.com/watch?v=k9ub2mNyd9M>
 They also talk about how to catalogue knots.

- (k) What is the Conway knot?
https://en.wikipedia.org/wiki/Conway_knot
- (l) Why two knots can't cancel each other?
Conway explains how he showed this in high school: <https://www.youtube.com/watch?v=lwWeRMmXIoU>
A reddit discussion on this issue: https://www.reddit.com/r/math/comments/9939yz/why_you_cant_untie_a_knot_by_tying_another_one/
- (m) Why are there no "usual" knots in 4d?
http://abel.math.harvard.edu/archive/21a_spring_06/exhibits/unknotted/index.html
- (n) What is high dimensional knot theory?
<https://www.youtube.com/watch?v=nYz3pRk1cCA> <https://www.maths.ed.ac.uk/~v1ranick/books/knot.pdf>
<https://arxiv.org/pdf/1304.6053.pdf>
<https://scholar.rose-hulman.edu/cgi/viewcontent.cgi?article=1292&context=rhumj>
- (o) 3d-printed models of knots presented by Laura Taalman:
<https://www.youtube.com/watch?v=YhXD7SR9EdQ>